Exercise 12

A crystal growth furnace is used in research to determine how best to manufacture crystals used in electronic components for the space shuttle. For proper growth of the crystal, the temperature must be controlled accurately by adjusting the input power. Suppose the relationship is given by

$$T(w) = 0.1w^2 + 2.155w + 20$$

where T is the temperature in degrees Celsius and w is the power input in watts.

(a) How much power is needed to maintain the temperature at 200°C?

(b) If the temperature is allowed to vary from 200°C by up to ±1°C, what range of wattage is allowed for the input power?

(c) In terms of the ε, δ definition of $\lim_{x\to a} f(x) = L$, what is x? What is $f(x)$? What is a? What is L? What value of ε is given? What is the corresponding value of δ?

Solution

Part (a)

Set $T = 200$ and solve the equation for w by using the quadratic formula.

$$200 = 0.1w^2 + 2.155w + 20$$

$$0 = 0.1w^2 + 2.155w - 180$$

$$w = \frac{-2.155 \pm \sqrt{2.155^2 - 4(0.1)(-180)}}{2(0.1)}$$

$$w \approx \{-54.5483, 32.9983\}$$

Since the power has to be positive, the negative result is discarded.

$$w \approx 32.9983 \text{ W}$$

Part (b)

With an allowed temperature variation of ±1°C, there are upper and lower bounds for the temperature.

$$\text{Lower Bound:} \quad 200 - 1 = 199°\text{C}$$

$$\text{Upper Bound:} \quad 200 + 1 = 201°\text{C}$$

Determine the power inputs corresponding to these temperatures.

$$199 = 0.1w_l^2 + 2.155w_l + 20 \quad \Rightarrow \quad w_l \approx \{\cancel{-54.4339}, 32.8839\} \approx 32.8839 \text{ W}$$

$$201 = 0.1w_u^2 + 2.155w_u + 20 \quad \Rightarrow \quad w_u \approx \{\cancel{-54.6624}, 33.1124\} \approx 33.1124 \text{ W}$$

Therefore, the allowed power variation is

$$\pm\frac{w_u - w_l}{2} \approx \pm\frac{1}{2}(33.1124 - 32.8839) \approx \pm 0.114225 \text{ W}.$$

Part (c)

In terms of the ε, δ definition of

$$\lim_{x \to a} f(x) = L,$$

x is the power w, $f(x)$ is the temperature $T(w)$, a is the power of about 32.9983 W corresponding to the desired 200°C temperature, L is the desired temperature 200°C, ε is the allowed temperature variation of 1°C, and δ is the allowed power variation of about 0.114225 W.